Answers to Questions 1-3

1. Question 1

Based on the free-space assumption that has preceded exploration of the mazes in project 1-3, all cells are assumed to be unblocked in the initial state (or at the very least, the agent has no reason to think any cell is more likely to be blocked over another). So, the agent starts with the number of cells assumed to be unblocked equal to the total number of cells in the maze.

Given this, the agent assumes that each cell is equally likely to contain the target. We can calculate this with the following:

P(ij containing target) = 1 / # of unblocked cells = **1 / # of total cells** \*\*for all cells i,j\*\*

1. Question 2

For all of the following equations, the LHS signifies what will become the probability that the target is contained within the associated cell at time t+1, and everything on the RHS is the current belief state of probabilities that cells contain the target at time t.

Each scenario will also show separate update equations for what we’ll call the “event” cell xy (i.e. the cell where the probability changes due to a discovery associated with that cell), and every other cell ij in the maze that is not equal to xy.

1. Update when attempting to enter a cell and finding it blocked

xy update

P(target is in xy | xy is blocked) = **0**

ij update

P(target is in ij | xy is blocked)

= P(target is in ij | target is not in xy)

// xy being blocked signifies that the cell doesn’t contain the target

= P(target in ij) \* P(target is not in xy | target in ij) / P(target is not in xy)

// P(target is not in xy | target in ij) is equal to 1 for fairly self-explanatory reasons

= P(target in ij) \* 1 / P(target is not in xy)

= **P(target in ij) / (1 – P(target is in xy))**

1. Update when entering a previously unvisited cell and discovering its terrain type

As discussed in the announcement posts by the professor, in real world situations, this would change the belief state, but for the purposes of this project, we are assuming that there is **no change** to the belief state in this situation.

1. Update when examining a cell with flat terrain and failing to find the target

xy update

P(target in xy | failed examination on xy of terrain type flat)

= P(target in xy) \* P(failed exam of flat xy | target in xy) / P(failed exam of flat xy)

// The false negative rate for flat terrain is 0.2

= P(target in xy) \* 0.2 / (1 – P(successful exam of flat xy))

// P(successful exam of flat xy) = ∑ P(target in α, successful exam of flat xy) via marginalization

// ∑ P(target in α, successful exam of flat xy)

// = ∑ P(target in α) \* P(successful exam of flat xy | target in α)

// = (some probability)\*0 + (some probability\*0) + … + P(Target in xy) \* 0.8 + …

// = P(Target in xy) \* 0.8

// The 0.8 multiplier comes from the probability of a true positive on flat terrain

= **0.2\*P(target in xy) / (1 – (0.8\*P(target in xy)))**

ij update

P(target in ij | failed examination of xy of terrain type flat)

= P(target in ij) \* P(failed exam of flat xy | target in ij) / P(failed exam of flat xy)

// Denominator is computed in the exact same way as in the xy update

// Also, P(failed exam of flat xy | target in ij) is equal to 1 because a failure is bound to happen

= P(target in ij) \* 1 / (1 – (0.8\*P(target in xy)))

= **P(target in ij) / (1 – (0.8\*P(target in xy)))**

1. Update when examining a cell with hilly terrain and failing to find the target

The derivations for the xy update and the ij update are identical to the corresponding updates completed in part c. The only difference in the final result is that the false negative rate for hilly terrain is 0.5, and the true positive rate for hilly terrain is 0.5.

xy update

P(target in xy | failed examination on xy of terrain type flat)

= **0.5\*P(target in xy) / (1 – (0.5\*P(target in xy)))**

ij update

P(target in ij | failed examination of xy of terrain type flat)

= **P(target in ij) / (1 – (0.5\*P(target in xy)))**

1. Update when examining a cell with forest terrain and failing to find the target

The derivations for the xy update and the ij update are identical to the corresponding updates completed in part c. The only difference in the final result is that the false negative rate for forest terrain is 0.8, and the true positive rate for forest terrain is 0.2.

xy update

P(target in xy | failed examination on xy of terrain type flat)

= **0.8\*P(target in xy) / (1 – (0.2\*P(target in xy)))**

ij update

P(target in ij | failed examination of xy of terrain type flat)

= **P(target in ij) / (1 – (0.2\*P(target in xy)))**

1. Update when target is found

xy update

P(target in xy | successful exam of xy) = **1**

ij update

P(target in ij | successful exam of xy) = **0**

1. Question 3

All of the following equations are based on the corresponding true positive rates.

We are assessing P(finding target in ij)

= P(successful exam of ij, target in ij)

= P(target in ij) \* P(successful exam of ij | target in ij).

P(successful exam of ij | target in ij) is a known probability that is equal to 1 – false negative rate for given terrain type.

1. Probability of finding the target in a cell that is hilly

P(finding target in ij)

= **0.5\*P(target in ij)**

1. Probability of finding the target in a cell that is flat

P(finding target in ij)

= **0.8\*P(target in ij)**

1. Probability of finding the target in a cell that is forest

P(finding target in ij)

= **0.2\*P(target in ij)**

1. Probability of finding the target in a cell that has never been visited

P(finding target in ij) = [(1/3)\*0.8\*P(target in ij) + (1/3)\*0.5\*P(target in ij) + (1/3)\*0.2\*P(target in ij)]

= (1/3)\*P(target in ij) \* (0.8 + 0.5 + 0.2) = (1/3)\*P(target in ij) \* (1.5)

**= 0.5\*P(target in ij)**